

On the field theoretical formulation of the electron-proton scattering in the Coulomb and Lorentz gauges.

A.I. Machavariani

Joint Institute for Nuclear Research, LIT Dubna, Moscow region 141980, Russia
and High Energy Physics Institute of Tbilisi State University, University str. 9
Tbilisi 380086, Georgia

Abstract

The relativistic three dimensional (3D) Lippmann-Schwinger-type equations for the ep scattering amplitude is derived based on unitarity condition in the usual quantum electrodynamic (QED). The ep scattering potential $V_{e'N',eN}$ consists of the leading one off mass shell photon exchange part and the nonlocal multi-particle exchange potential. Unlike to the other field-theoretical equations, both protons in the unitarity condition and in $V_{e'N',eN}$ are on mass shell. Therefore in this approach are not required the multi-variable input photon-nucleon vertexes with the off mass shell nucleons.

In the present formulation the standard leading one photon exchange potential V_{OPE} is generated by the canonical equal-time anti commutator between the electron source and the interacted electron fields which are sandwiched by the one nucleon asymptotic states. This anticommutator is calculated in the Coulomb and Lorentz gauges, where only the transverse parts of the photon fields are quantized. It is shown, that the leading one photon exchange potential V_{OPE} in the Coulomb and Lorentz gauges coincide. The complete set of the next to leading order terms which are generated by the static electric (Coulomb) interaction are exactly reproduced.

1. Introduction

Presently the elastic ep scattering in energy region up to few GeV is well described by the leading order one photon exchange (OPE) model. In this energy region the various observables for the ep scattering reactions were studied within the next of leading order two photon exchange (TPE) models in order to check the accuracy of the OPE model. The OPE model was satisfactorily justified by description of the ratio of the electric and magnetic form factors of the proton, angular dependence of the polarisation observables, electro-production of the pions and resonances, the ratio of the cross sections of the e^-p and e^+p reactions, electromagnetic form factors of the neutron and other observables in the high precision experiments [1, 2, 3]. The important uncertainty in the calculations of the ep scattering within the TPE models arise from off shell extension of the γ^*pp vertexes [4] because the loop diagrams contain the γ^*pp vertexes with off mass shell nucleons which depends on the two or three variables. Therefore there was used the additional models in order to construct or simulate the multi-variable γ^*pp form factors.

An other reason for investigation of the high order power series expansion in e^2 of the ep scattering amplitude is that 60 years ago Dyson argued (see [7], ch. 9.4 in [5] and ch. 20.7 in [6]) that the perturbation series in QED are divergent after renormalization of the mass and charge. Therefore, there is necessary to use the special methods for calculations of the two-photon and multi-photon exchange effects. For instance, the perturbative series in QED can be calculated via the special asymptotic expansions [8] or using the mathematical methods without power series expansion in e^2 . One of the non perturbative approaches based on the relativistic field theoretical generalization of the Lippmann-Schwinger equations $A = V + VG_oA$, where one can determine $(G_o - V)^{-1}$ and the full ep scattering amplitude $A = V(1 - G_oV)^{-1}$ even if the perturbative series $A = V + VG_oV + VG_oVG_oV + \dots$ are divergent.

The 3D quasipotential reductions of the 4D Bethe-Salpeter equation for the Hydrogen-type systems are used in [9, 10, 11]. In these approaches the 3D relativistic Lippmann-Schwinger type equations were applied to the high order perturbation terms for the energy levels. But the calculation are performed based on the eigenfunctions of the Schrödinger equation with the Coulomb potential or the eigenfunctions of the Dirac equation in the external Coulomb field. The solution of the relativistic Dirac equation was applied also for calculation of the ion-atom scattering [12].

This paper deals with the new kind 3D Lippmann-Schwinger-type field-theoretical equations for the ep scattering amplitude $A_{e'N',eN}$ within the usual quantum electrodynamic (QED), where the protons in the corresponding potential $V_{e'N',eN}$ are on mass shell. These 3D Lippmann-

Schwinger equation allow to build ep discrete spectrum and the ep elastic scattering amplitudes based on the same potentials. The present approach based on the 3D field-theoretical unitarity condition and is free from the 3D ambiguities which appears by the 3D (quasipotential) reductions of the 4D Bethe-Salpeter equations. Unlike to the 4D Bethe-Salpeter equations and their 3D quasipotential reductions, the potential of the suggested equations is constructed from the one-variable vertexes with on mass shell protons. The off mass shell intermediate particle exchange part in the suggested approach is generated by the canonical equal time anti commutators which is sandwiched by the asymptotic one proton states. This part of the potential generates the leading OPE potential. These equal-time canonical anti commutators we shall derive in the Coulomb and Lorentz gauges.

The considered method was firstly applied to the low energy πN scattering by Chew and Low [23] and was developed by Banerjee and coworkers [24]. The analytical model-independent linearization of these relativistic quadratically nonlinear equations was done in [13, 14, 15], where they were applied to descriptions of the low energy πN and NN phase shifts. The leading order one-particle exchange diagrams in this approach for the multichannel reactions $ep - e'p'\gamma'$ and $\gamma p - \gamma.\pi'p$ in the Δ -resonance region was studied in [16, 17, 18, 19].

The present paper consists of the five sections. In the next section the general unitarity condition for the ep scattering amplitudes with the on mass shell protons and the corresponding 3D Lippmann-Schwinger type equation are considered. The calculation of the OPE potential through the equal time canonical anti commutators in the Coulomb gauge is performed in section 3. In the section 4 the same anti commutators and the potentials are calculated in the Lorentz gauge and the exact relationship of these expressions in the Lorentz and in the Coulomb gauges are suggested. The Summary is given in section 5.

2. Unitarity condition and corresponding 3D relativistic equation within the time ordered relativistic field-theoretical approach

In the quantum field theory [5, 20] the elastic ep scattering amplitude is

$$\mathcal{A}_{e'N',eN} = - < out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_e \mathbf{p}_N; in > \quad (2.1a)$$

where $(p_e^o \equiv E_{\mathbf{p}_e} = \sqrt{\mathbf{p}_e^2 + m_e^2}, \mathbf{p}_e)$, $(p_N^o \equiv E_{\mathbf{p}_N} = \sqrt{\mathbf{p}_N^2 + m_N^2}, \mathbf{p}_N)$ and $(p_e'^o \equiv E_{\mathbf{p}_e'} = \sqrt{\mathbf{p}_e'^2 + m_e^2}, \mathbf{p}_e')$, $(p_N'^o \equiv E_{\mathbf{p}_N'} = \sqrt{\mathbf{p}_N'^2 + m_N^2}, \mathbf{p}_N')$ denote the energy-momentum of the on mass shell electron and proton in the initial and final states. $\left(i\gamma_\mu \partial/\partial x_\mu - m_e\right)\psi_e(x) = \eta_e(x)$ stand

for the source operators of the electron, and

$$\eta_{\mathbf{p}'_e}(x) = \bar{u}(\mathbf{p}'_e)\eta(x) \quad (2.1b)$$

The normalizations and designations of the symbols in above expressions are taken from [5]. For the sake of simplicity the spin indexes in the expressions (2.1a,b) and in the Dirac spinor $u(\mathbf{p}_e)$ are omitted.

In the amplitude (2.1a) the final electron is extracted from the asymptotic "out" states and it depends on \mathbf{p}'_e only through $\bar{u}(\mathbf{p}'_e)$. Therefore, the 4-momenta of the final electron e' can be chosen in such a way that e' is on energy shell but off mass shell, i.e. $(p'_e)_\mu = (p_e)_\mu + (p_N)_\mu - (p'_N)_\mu$ and $(p'_e)^2 \neq m_e^2$. According to the reduction formulas [20] the ep scattering amplitude can be represented as

$$\mathcal{A}_{e'N',eN} = \langle out; \mathbf{p}'_N | \left(\left\{ \eta_{\mathbf{p}'_e}(0), b_{\mathbf{p}_e}^+(0) \right\} - i \int d^4x e^{-ip_e x} \theta(-x_o) \left\{ \eta_{\mathbf{p}'_e}(0), \bar{\eta}_{\mathbf{p}_e}(x) \right\} \right) | \mathbf{p}_N; in \rangle, \quad (2.2)$$

where the curly braces denote the anticommutator of the corresponding operators, $b_{\mathbf{p}_e}^+(x_o)$ transforms into corresponding creation or annihilation operator in the asymptotic $out(in)$ region $\lim_{x_o \rightarrow \pm\infty}^{weakly} b_{\mathbf{p}_e}^+(x_o) \rightarrow b_{\mathbf{p}_e}^+(out(in))$ and

$$b_{\mathbf{p}_e}^+(x_o) = \int d^3x e^{-iE_{\mathbf{p}_e}x_o + i\mathbf{p}_e\mathbf{x}} \bar{\psi}_e(x) \gamma_o u(\mathbf{p}_e); \quad b_{\mathbf{p}_e}^{o+}(x_o) \equiv \frac{\partial b_{\mathbf{p}_e}^+(x_o)}{\partial x_o} = i \int d^3x e^{-iE_{\mathbf{p}_e}x_o + i\mathbf{p}_e\mathbf{x}} \bar{\eta}_{\mathbf{p}_e}(x). \quad (2.3)$$

For derivation of (2.2) the identity $b_{\mathbf{p}_e}^+(in) = b_{\mathbf{p}_e}^+(0) - \int dx_o \theta(-x_o) b_{\mathbf{p}_e}^{o+}(x_o)$ was used.

Substitution the completeness condition of the asymptotic in-states

$$\sum_n |n; in \rangle \langle in; n| = 1 \quad (2.4)$$

between the source operators in (2.2) and integration over x yields

$$\begin{aligned} & - \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_e \mathbf{p}_N; in \rangle = \langle out; \mathbf{p}'_N | \left\{ \eta_{\mathbf{p}'_e}(0), b_{\mathbf{p}_e}^+(0) \right\} | \mathbf{p}_N; in \rangle \\ & + \sum_{n=H, ep, ep\gamma, \dots} \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | n; in \rangle \frac{(2\pi)^3 \delta(\mathbf{P}_n - \mathbf{p}_e - \mathbf{p}_N)}{P_o - P_n^o + i\epsilon} \langle in; n | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle \\ & + \sum_{m=\bar{e}p, \bar{e}\gamma p, \dots} \langle out; \mathbf{p}'_N | \bar{\eta}_{\mathbf{p}_e}(0) | m; in \rangle \frac{(2\pi)^3 \delta(\mathbf{P}_m + \mathbf{p}_e - \mathbf{p}'_N)}{E_{\mathbf{p}_e} - E_{\mathbf{p}'_N} + P_m^o} \langle in; m | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_N; in \rangle, \quad (2.5) \end{aligned}$$

where $P_o = E_{\mathbf{p}_e} + E_{\mathbf{p}_N}$ and $P'_o = E_{\mathbf{p}'_e} + E_{\mathbf{p}'_N}$ are the energies of the ep system in the initial and final states, P_n^o and \mathbf{P}_n stand for the energy and momentum of the intermediate n -particle states, H and \bar{e} denote the on mass shell intermediate ep bound state (Hydrogen) and positron.

If all of the fourth components of the photon field $A_\mu(x)$ are independent and quantized, then we have the following canonical commutation relations for the interacted electron and photon fields [5]

$$\left[\frac{\partial A_\mu(x_o, \mathbf{x})}{\partial x_o}, A_\nu(x_o, \mathbf{y}) \right] = ig_{\mu\nu} \delta(\mathbf{x} - \mathbf{y}) \quad (2.6a)$$

$$\left[A_\mu(x_o, \mathbf{x}), A_\nu(x_o, \mathbf{y}) \right] = 0; \quad \left[\psi(x_o, \mathbf{x}), A_\nu(x_o, \mathbf{y}) \right] = 0; \quad (2.6b)$$

$$\left\{ \psi(x_o, \mathbf{x}), \psi^\dagger(x_o, \mathbf{y}) \right\} = \delta(\mathbf{x} - \mathbf{y}) \quad (2.6c)$$

that allow to determine the first term in (2.2)

$$Y_{e'N',eN} = \langle out; \mathbf{p}'_N | \left\{ \eta_{\mathbf{p}'_e}(0), b_{\mathbf{p}_e}^\dagger(0) \right\} | \mathbf{p}_N; in \rangle \quad (2.7a)$$

as the usual leading one photon exchange potential

$$Y_{e'N',eN} = V_{OPE} = e \bar{u}(\mathbf{p}'_e) \gamma_\mu u(\mathbf{p}_e) \frac{1}{t_N} \langle out; \mathbf{p}'_N | J^\mu(0) | \mathbf{p}_N; in \rangle, \quad (2.7b)$$

where $t_N = (p_N'^0 - p_N^0)^2 - (\mathbf{p}'_N - \mathbf{p}_N)^2$ and $J^\mu(x)$ denotes the photon source

$$J^\mu(x) = e \bar{\psi}_e(x) \gamma^\mu \psi_e(x) + e \bar{\psi}_N(x) \gamma^\mu \psi_N(x) + \dots \quad (2.8)$$

where ... stands for the sources of the intermediate particles.

The $\gamma^* p'p$ -vertex $\langle out; \mathbf{p}'_N | J^\mu(0) | \mathbf{p}_N; in \rangle$ in (2.7b) with the off mass shell proton is determined via the usual Dirac electric and magnetic form factors of the proton

$$\langle out; \mathbf{p}'_N | J^\mu(0) | \mathbf{p}_N; in \rangle = \bar{u}(\mathbf{p}'_N) \left(\gamma^\mu F_1(t_N) + i \frac{\sigma^{\mu\nu} (p' - p)_\nu}{2m_N} F_2(t_N) \right) \quad (2.9)$$

The graphical representation of the one off mass shell photon exchange potential V_{OPE} (2.7b) is given in Fig. 1. In this diagram the shaded circle corresponds to the $\gamma^* NN$ vertex (2.9) and the dot between the electron-photon lines relates to the $\gamma^* ee$ vertex in the tree approximation.

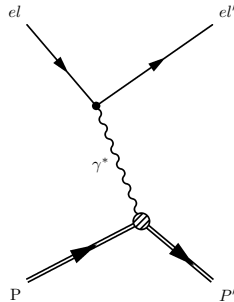


Figure 1: The t -channel one photon exchange term (2.7b) with off mass shell proton.

The canonical commutation relations (2.6a,b) requires the the indefinite metric according to Gupta-Bleuer approach[5]. In order to avoid this complication in the next sections we shall

consider the Lorentz and Coulomb gauges where only the transversely parts of the photon fields are independent and quantized.

The vertexes of the transition in the intermediate n and m particle states in (2.5) consists of the connected and disconnected parts. The cluster decomposition procedure [22, 24] allow to represent the particle exchange paths in (2.5) through the connected vertexes. Then the second and third terms in (2.5) generate the eight expression (A.1a)-(A.1h) in Appendix A with $n = H, ep, ep\gamma, \dots$. The on mass shell particle exchange terms (A.1a)-(A.1h) with the simplest two and three body intermediate states are depicted in Fig. 2. These terms have different chronological sequences of the absorption and emission of the external electrons and nucleons. In particular, the s -channel diagram in Fig. 1A the initial eN state transfers into intermediate $\gamma e'' N''$ state which afterwards transforms into final $e' N'$ state. In Fig. 1B the photon-electron intermediate state $\gamma e''$ is generated by the $eN \rightarrow \gamma e'' N$ transition amplitude and after truncation of the photon $\gamma e''$ transforms into e' . In Fig.1C firstly electron radiate photon and afterwards the intermediate $\gamma e''$ state is truncated via the amplitude $\gamma e'' N \rightarrow e' N'$. The diagram in Fig. 1D describes the chain of reactions, where firstly the initial electron generates the $e'' \bar{N} N'$ state with the final nucleon N' and afterwards this intermediate state together with the initial nucleon N produces the final electron e' . The u -channel terms in the diagrams in Fig. 1E,F,G,H are obtained from the s -channel terms in Fig. 1A,B,C,D via crossing of the external electrons. Therefore, in Fig. 1E,F,G,H firstly is radiated the final electron and afterwards is truncated the initial electron. Consequently the chine of the reactions in Fig. 1E,F,G,H contains the intermediate anti-electron (positron) state \bar{e} . Thus any s -channel diagram have the corresponding antiparticle exchange so called Z diagrams [22].

The particle in the intermediate states in the expressions (A.1a)-(A.1h) are on mass shell. Therefore unlike to the 4D formulations in (A.1a)-(A.1h) do not contain the energy and charge renormalization diagrams and proton vertex correction diagrams.

It is convenient to consider separately the s -channel exchange terms with the intermediate states $n = H, ep$

$$\mathcal{A}_{e'N',eN} = Y_{e'N',eN} + \mathcal{W}_{e'N',eN} + \sum_H \mathcal{A}_{e'N',H} \frac{1}{P_o - P_H^o} \mathcal{A}_{H,eN}^+ + \sum_{e''N''} \mathcal{A}_{e'N',e''N''} \frac{1}{P_o - P_{e''N''}^o + i\epsilon} \mathcal{A}_{e''N'',eN}^+, \quad (2.10)$$

where $\mathcal{W}_{e'N',eN}$ is determined in (A.1a)-(A.1h) without on shell $n = H, ep$ exchange terms.

It is easy to check, that the ep scattering amplitude $\mathcal{A}_{e'N',eN}$ in (2.5) and in (2.10) satisfies

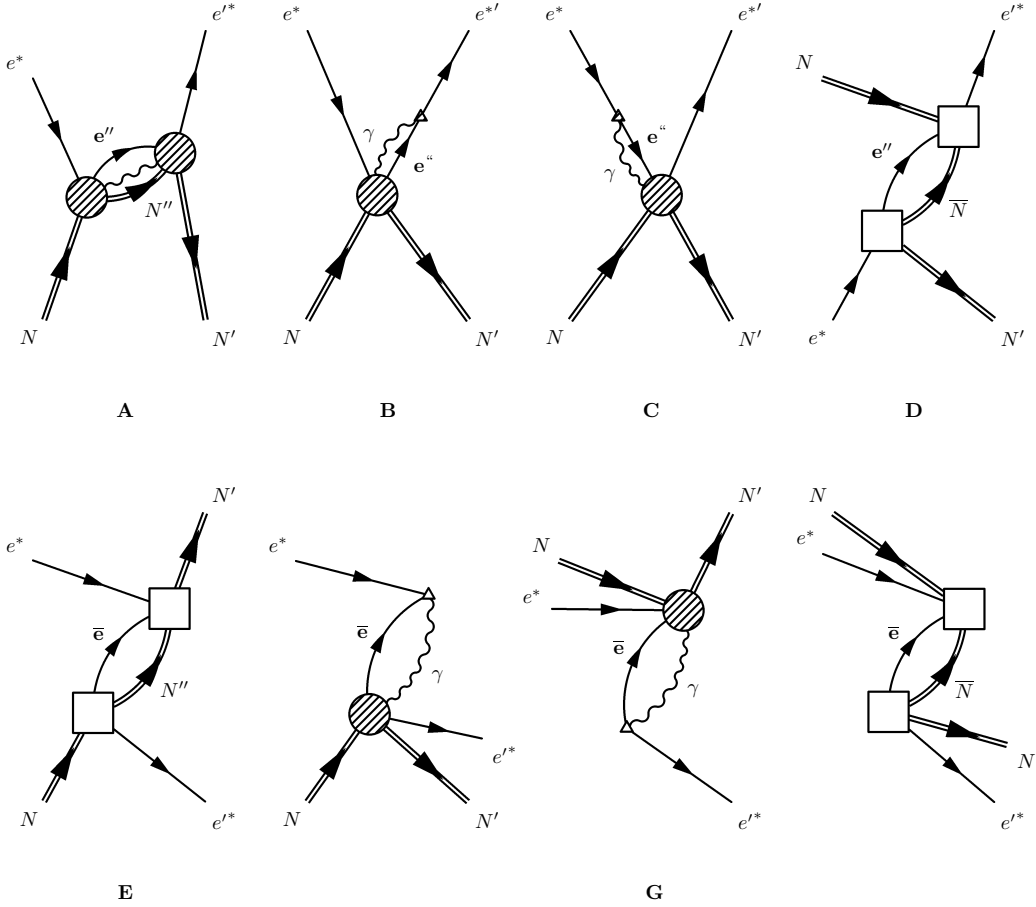


Figure 2: On mass shell particle exchange time ordered diagrams with the off mass shell electrons. The dashed circle denotes the five particle $ep \leftrightarrow ep\gamma$ transition amplitudes and their crossings. Square stands for the four point amplitudes $ep \leftrightarrow ep$ and their crossings. (D) and (H) presents the diagrams with the intermediate anti nucleon. (E), (F), (G) and (H) are the u-channel diagrams with the intermediate anti electron exchange.

the general unitarity condition with all possible n -particle s -channel intermediate states

$$\mathcal{A}_{e'N',eN} - \mathcal{A}_{e'N',eN}^+ = \sum_{n=ep,ep\gamma,\dots} \mathcal{A}_{e'N',n} \left[\frac{(2\pi)^3 \delta(\mathbf{P}_n - \mathbf{p}_e - \mathbf{p}_N)}{P_o - P_n^o + i\epsilon} - \frac{(2\pi)^3 \delta(\mathbf{P}_n - \mathbf{p}'_e - \mathbf{p}'_N)}{P'_o - P_n^o - i\epsilon} \right] \mathcal{A}_{en,n}^* \quad (2.11)$$

The analytic linearization of the 3D time-ordered and quadratically nonlinear equation (2.10) can be performed in the same way as for the elastic πN or NN scattering [13, 14, 15] or for the reaction $\pi N - \gamma' N'$ [16]-[19]. The resulting relativistic Lippmann-Schwinger type is

$$\mathcal{T}(\mathbf{p}', \mathbf{p}, P_o) = \mathcal{U}(\mathbf{p}', \mathbf{p}, P_o) + \int \mathcal{U}(\mathbf{p}', \mathbf{q}, P_o) \frac{d^3 \mathbf{q}}{P_o - E_{\mathbf{p}'_e} - E_{\mathbf{p}'_N} + i\epsilon} \mathcal{T}(\mathbf{q}, \mathbf{p}, P_o) \quad (2.13)$$

where the on shell $\mathcal{T}(\mathbf{p}', \mathbf{p}, P_o)$ coincides with the on shell amplitude $\mathcal{A}_{e'N',eN}$ (2.1a)

$$\left[\mathcal{T}(\mathbf{p}', \mathbf{p}, P_o) = - \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_e \mathbf{p}_N; in \rangle \right]^{on \quad shell}, \quad (2.14)$$

if $\mathcal{U}(\mathbf{p}', \mathbf{p}, P_o)$ is constructed through the inhomogeneous term of (2.10) as

$$\mathcal{U}(\mathbf{p}', \mathbf{q}, E) = Y_{e'N',eN}(\mathbf{p}', \mathbf{p}) + \mathbf{A}_{e'N',eN}(\mathbf{p}', \mathbf{p}) + E \mathbf{B}_{e'N',eN}(\mathbf{p}', \mathbf{p}) \quad (2.15a)$$

where

$$\mathcal{W}_{e'N',eN}(\mathbf{p}', \mathbf{p}) = \mathbf{A}_{e'N',eN}(\mathbf{p}', \mathbf{p}) + P_o' \mathbf{B}_{e'N',eN}(\mathbf{p}', \mathbf{p}), \quad (2.15b)$$

The exact form of $\mathcal{W}_{e'N',eN}$, $\mathbf{A}_{e'N',eN}$ and $\mathbf{B}_{e'N',eN}(\mathbf{p}', \mathbf{p})$ is given in Appendix A,

In the half off energy region $\mathcal{A}_{e'N',eN}$ (2.1a) is determined through $\mathcal{T}(\mathbf{p}', \mathbf{p}, P_o')$ as

$$- \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_e \mathbf{p}_N; in \rangle = \mathcal{U}(\mathbf{p}', \mathbf{q}, P_o') + \int \mathcal{U}(\mathbf{p}', \mathbf{q}, P_o') \frac{d^3 \mathbf{q}}{P_o - E_{\mathbf{p}'_e} - E_{\mathbf{p}'_N} + i\epsilon} \mathcal{T}(\mathbf{q}, \mathbf{p}, P_o) \quad (2.16)$$

where $P_o' = E_{\mathbf{p}'_e} + E_{\mathbf{p}'_N}$.

Thus the solution of the 3D relativistic Lippmann-Schwinger-type equation (2.13) determine the eN scattering amplitude $- \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_e \mathbf{p}_N; in \rangle$ (2.1a) according to (2.14) and (2.16).

The \mathcal{S} -matrix reduction formulas [5, 20] allow to reproduce the amplitude (2.1a) from the full Green function $\langle 0 | T(\psi_e(0) \psi_N(x) \bar{\psi}_e(y_e) \bar{\psi}_e(y_N)) | 0 \rangle$ and the corresponding 4D Bethe-Salpeter amplitude. Unlike to the 4D Bethe-Salpeter equations and their 3D reductions in the field-theoretical 3D equation (2.13) as input appear the leading OPE potential (2.7b) with the usual one-variable form factors (2.9). Besides the solution of the 3D equation (2.13) satisfy the unitarity condition even after truncation of the intermediate on mass shell particle states in (A.1a)-(A.1h). In the unitarity conditions (2.10) and the corresponding 3D Lippmann-Schwinger type equations nucleons are on mass shell. Therefore the quark-gluon degrees of freedom can contribute only through the input OPE potential (2.7b) and the form factors (2.9).

3. Canonical commutation relations and the Born term (2.7a) in the Coulomb gauge.

In the Coulomb gauge

$$\frac{\partial A_i^C(x)}{\partial x_i} = 0; \quad i = 1, 2, 3 \quad (3.1)$$

as independent are considered the electron fields $\psi_e(x)$, $\psi_e^+(x)$ and the transverse components of the photon field

$$A_i^C \equiv A_i^{tr} = A_i - \frac{1}{\Delta} \frac{\partial A_i}{\partial x_i}; \quad \Delta = \frac{\partial^2}{\partial x_i \partial x_i} \quad (3.2)$$

The conserved current $J^\nu(x)$ (2.8) and the gauge condition (3.1) yields the equations of motion [5, 20, 21]

$$\square_x A^C_i(x) = J^{tr}_i(x) = J_i(x) - \frac{\partial}{\partial x^i} \frac{\partial J^k(x)}{\partial x^k} \quad (3.3)$$

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e\right)\psi_e(x) = e\gamma^\mu A^C_\mu(x)\psi_e(x) \quad (3.4)$$

where the zero component of the photon field A^C_o is defined via the photon source (2.8) according to the Poisson equation

$$-\Delta A^C_o(x) \equiv -\frac{\partial}{\partial x^i} \frac{\partial A^C_o(x)}{\partial x_i} = J^o_i(x); \quad A^C_o(x) = \int \frac{d\mathbf{x}' J_o(x_o, \mathbf{x}')}{4\pi|\mathbf{x} - \mathbf{x}'|} \quad (3.5)$$

The equation of motion (3.3) does not contain the longitudinal part of the A^C_i in virtue of (3.1). Nevertheless in the electromagnetic Lagrangian

$$\mathcal{L}_{em}^{(C)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^{tr}_i(A^C)^i \quad (3.6)$$

with $F_{\mu\nu}(x) = \partial A^\nu(x)/\partial x^\mu - \partial A^\mu(x)/\partial x^\nu$ the longitudinal part of the electric field $\mathbf{E}^l_i = -\partial A^C_o/\partial x_i$ appears through the complete electric fields

$$\mathbf{E}_i = F_{io} = \mathbf{E}^{tr}_i + \mathbf{E}^l_i = -\frac{\partial A_o}{\partial x^i} - \frac{\partial A_i}{\partial x_o}; \quad (\mathbf{E}^{tr})_i = -\frac{\partial A^C_i}{\partial x_o} \quad (3.7)$$

Therefore one can separate the electrostatic (Coulomb) interaction term $J^o A^C_o$ in (3.6) [20, 21]

$$\mathcal{L}_{em}^{(C)} = -\frac{1}{2}(\mathbf{E}^{tr})^2 - \frac{1}{2}\mathbf{B}^2 - J^{tr}_i A^C_i + J^o A^C_o \quad (3.8a)$$

where $\mathbf{B} = -rot\mathbf{A}^C$ and $J^o A^C_o$ was generated by \mathbf{E}^l_i according to the relation

$$\int d^3\mathbf{x}(\mathbf{E}^l)^2 = \int d^3\mathbf{x}A^C_o J^o \quad (3.8b)$$

The canonical commutation relations for A^C_i [20, 21] are

$$\left[\frac{\partial A^C_i(x_o, \mathbf{x})}{\partial x_o}, A^C_j(x_o, \mathbf{y})\right] = \delta_{ij}\delta(\mathbf{x} - \mathbf{y}) - \frac{1}{\Delta} \frac{\partial^2}{\partial x_i \partial y_j} \frac{1}{|\mathbf{x} - \mathbf{y}|} \quad (3.9)$$

The source $\eta^C = e\gamma^o A^C_o \psi_e - e\gamma^i A^C_i \psi$ in (3.4) and the canonical commutators (2.6c) allow to represent the Born term (2.7a) as

$$Y_{e'N', eN}^C \equiv Y_I^C - Y_{II}^C = \quad (3.10a)$$

where

$$Y_I^C = e\bar{u}(\mathbf{p}'_e)\gamma^\mu <out; \mathbf{p}'_N|A^C_\mu(0)\left\{\psi_e(0), b_{\mathbf{p}_e}^+(0)\right\}|\mathbf{p}_N; in > = e\bar{u}(\mathbf{p}'_e)\gamma^\mu u(\mathbf{p}_e) <out; \mathbf{p}'_N|A^C_\mu(0)|\mathbf{p}_N; in > \quad (3.10b)$$

$$Y_{II}^C = -e \langle out; \mathbf{p}'_N | \left[A_\mu^C(0), b_{\mathbf{p}_e}^+(0) \right] \psi_e(0) | \mathbf{p}_N; in \rangle = -e \bar{u}(\mathbf{p}'_e) \gamma^o \int \frac{d\mathbf{x}'}{4\pi|\mathbf{x}'|} \langle out; \mathbf{p}'_N | \psi_e^+(0, \mathbf{x}') \psi_e(0, 0) | \mathbf{p}_N; in \rangle u(\mathbf{p}_e) \quad (3.10c)$$

The Poisson equation (3.5) and the equation of motion (3.4) yield

$$Y_I^C = \frac{e \bar{u}(\mathbf{p}'_e) \gamma^o u(\mathbf{p}_e)}{-(\mathbf{p}'_N - \mathbf{p}_N)^2} \langle out; \mathbf{p}'_N | J_o^{tr}(0) | \mathbf{p}_N; in \rangle - e \bar{u}(\mathbf{p}'_e) \gamma^i u(\mathbf{p}_e) \frac{1}{t_N} \langle out; \mathbf{p}'_N | J_i^{tr}(0) | \mathbf{p}_N; in \rangle \quad (3.11)$$

The longitudinal part of A_i^C and J_i are excluded from Y_I^C (3.11). Nevertheless in the next section we shall demonstrate that Y_I^C (3.11) coincides with V_{OPE} (2.7b).

Y_{II}^C (3.10c) is nonlocal due to integration of the nonlocal source $e \psi_e^+(0, \mathbf{x}') \psi_e(0, 0)$. Insertion of the completeness condition (2.4) between the electron fields in (3.10c) yields

$$Y_{II}^C = -e \sum_{n=e''N, e''N\gamma, \dots} \frac{\langle out; \mathbf{p}'_N | \bar{\eta}_{\mathbf{p}'_e}(0) | n; in \rangle_C}{(\mathbf{p}'_N - \mathbf{p}_n)^2 (i\gamma_\sigma (p'_N - P_n)^\sigma - m_e)} \frac{\langle in; n | \eta_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle}{(i\gamma_\nu (-p_N + P_n)^\nu - m_e)} \quad (3.12a)$$

$$+e \sum_{m=\gamma N, \gamma\gamma N, \dots} \frac{\langle 0 | \bar{\eta}_{\mathbf{p}'_e}(0) | m; in \rangle_C \langle in; \mathbf{p}'_N, m | \eta_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle}{\mathbf{p}_m^2 (i\gamma_\sigma P_m^\sigma - m_e)} \frac{\langle in; \mathbf{p}'_N, m | \eta_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle}{(i\gamma_\nu (p'_N - p_N - P_m)^\nu - m_e)} \quad (3.12b)$$

$$+e \sum_{m=\gamma N, \gamma\gamma N, \dots} \frac{\langle out; \mathbf{p}'_N | \bar{\eta}_{\mathbf{p}'_e}(0) | m, \mathbf{p}_N; in \rangle_C}{(\mathbf{p}'_N - \mathbf{p}_N - \mathbf{p}_m)^2 (i\gamma_\sigma (p'_N - p_N - P_m)^\sigma - m_e)} \frac{\langle in; m | \eta_{\mathbf{p}_e}(0) | 0 \rangle}{(i\gamma_\nu P_m^\nu - m_e)} \quad (3.12c)$$

$$-e \sum_{\bar{n}=e''\bar{N}, e''\bar{N}\gamma, \dots} \frac{\langle 0 | \bar{\eta}_{\mathbf{p}'_e}(0) | \mathbf{p}_N \bar{n}; in \rangle_C}{(-\mathbf{p}_N - \mathbf{p}_{\bar{n}})^2 (i\gamma_\sigma (-p_N - P_{\bar{n}})^\sigma - m_e)} \frac{\langle in; \mathbf{p}'_N, \bar{n} | \eta_{\mathbf{p}_e}(0) | 0 \rangle}{(i\gamma_\nu (p'_N + P_{\bar{n}})^\nu - m_e)} \quad (3.12d)$$

These terms have the same structure as the on mass shell particle exchange amplitudes (A.1a) - (A.1d) correspondingly. The terms (3.12a)-(3.12d) consists of the products of the ep scattering matrices and corresponding crossing terms, i.e. these terms give e^5 contributions into ep scattering amplitude. The nonlocality of (3.12a)-(3.12d) follows from the canonical equal time commutations between $b_{\mathbf{p}}^+(0)$ and the nonlocal field $A_o^C(0)$.

4. The Born term (2.7a) in the Lorentz and Coulomb gauges.

The Lorentz gauge

$$\frac{\partial A_\mu^L(x)}{\partial x_\mu} = 0 \quad (4.1)$$

allows to formulate the ep scattering problem in the Lorentz invariant 4D form if the four components of the photon field A_μ^L are the independent variables. In order to avoid the problems with the indefinite metric in the Gupta-Bleuer formalism and simplify comparison with the

relations in the Coulomb gauge, we consider in the Lorentz gauge as independent fields electrons and the transverse photons. Consequently we keep the canonical equal time anti commutation (2.6c) for the electrons and (3.9) for the transverse part of A^L_μ . Then in the Lorentz gauge we get the following equation of motion

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m_e\right)\psi_e(x) = \eta(x) = e\gamma^\mu A^L_\mu(x)\psi_e(x) \quad (4.2a)$$

$$\square_x(A^L)_i^{tr}(x) = J_i^{tr}(x) = J_i(x) - \frac{\partial}{\partial x^i} \frac{\partial J^k(x)}{\partial x^k}; \quad i = 1, 2, 3 \quad (4.2b)$$

and the zeroth and longitudinal components of the photon fields are determined through the corresponding sources as

$$(A^L)_i^l(x) = \square_x^{-1} J_i^l(x); \quad J_i^l(x) = e \frac{\partial}{\partial x^i} \frac{\partial [\bar{\psi}_e(x) \gamma_k \psi_e(x)]}{\partial x_k} \quad (4.2c)$$

$$A^L_o(x) = \square_x^{-1} J_o(x); \quad J_o(x) = e \bar{\psi}_e(x) \gamma_o \psi_e(x), \quad (4.2d)$$

where

$$(A^L)_i^{tr}(x) = A^L_i(x) - \frac{\partial}{\partial x_i} \frac{\partial A^L_k(x)}{\partial x_k}; \quad (A^L)_i^l(x) = \frac{\partial}{\partial x^i} \frac{\partial (A^L)_k(x)}{\partial x_k} \quad (4.3)$$

The gauge condition (4.1) allows to redefine $(A^L)_i^l$ through the A^L_o

$$(A^L)_i^l(x) = \frac{\partial}{\partial x^i} \frac{\partial A^L_o(x)}{\partial x^o} \quad (4.4)$$

Thus we have two auxiliary fields $(A^L)_i^l$ and A^L_o in the Lorentz gauge (4.1). Both of these auxiliary fields are determined through the J_o according to (4.2c,d) and (4.4).

The first part of the equal time commutator (2.7a) $Y^l = Y_I^L - Y_{II}^L$ in the Lorentz gauge

$$Y_I^L = e \bar{u}(\mathbf{p}'_e) \gamma^\mu < out; \mathbf{p}'_N | A^L_\mu(0) \left\{ \psi_e(0), b_{\mathbf{p}_e}^+(0) \right\} | \mathbf{p}_N; in > = e \bar{u}(\mathbf{p}'_e) \gamma^\mu u(\mathbf{p}_e) < out; \mathbf{p}'_N | A^L_\mu(0) | \mathbf{p}_N; in > \quad (4.5)$$

reproduces exactly the Born term V_{OPE} (2.7b).

The next part of the equal-time commutators (2.7a)

$$Y_{II}^L = -e \bar{u}(\mathbf{p}'_e) \gamma^{\mu=0,3} < out; \mathbf{p}'_N | \left[A^L_{\mu=0,3}(0), b_{\mathbf{p}_e}^+(0) \right] \psi_e(0) | \mathbf{p}_N; in > \quad (4.6)$$

is more complicated than Y_{II}^C (3.10c) because $A^L_{\mu=0,3}$ in (4.2d) contains additional integration over the x'_o of the sources $J_o(x')$.

Thus Y_I^L (4.5) and Y_I^C (3.11) determine the leading one photon exchange (OPE) term in the Lorentz and Coulomb gauge correspondingly. These OPE potentials (Fig. 1) are constructed via the nucleon vertexes

$$F_\mu^L = \langle out; \mathbf{p}'_N | A_\mu^L(0) | \mathbf{p}_N; in \rangle \quad (4.7)$$

$$F_\mu^C = \langle out; \mathbf{p}'_N | A_\mu^C(0) | \mathbf{p}_N; in \rangle. \quad (4.8)$$

The photon fields in the Lorentz and Coulomb gauges $A_\mu^L(x)$ and $A_\mu^C(x)$ can be obtained independently through the gauge transformation in the Dirac equation for the noninteracting electrons. In order to determine the relationship between the vertexes (4.7) and (4.8) we consider the gauge transformations between these fields

$$\Psi^C(x) = e^{i\lambda(x)} \Psi^L(x); \quad A_\mu^C(x) = e^{-i\lambda(x)} A_\mu^L(x) e^{i\lambda(x)} + \frac{\partial \lambda(x)}{\partial x_\mu} \quad (4.9)$$

where the electron field ψ_e is denoted as Ψ^L and Ψ^C in the Lorentz and Coulomb gauges.

Generally $\lambda(x)$ contain the auxiliary fields $A_{\mu=3,0}^L$ which does not commute with Ψ^L . Therefore, it is convenient to extract from A_μ^C the part

$$\mathcal{D}_\mu(x) \equiv e^{-i\lambda(x)} A_\mu^L(x) e^{i\lambda(x)} - A_\mu^L(x) = \left[A_{\mu=0,3}^L(x), \lambda(x) \right] + \left[\left[A_{\mu=0,3}^L(x), \lambda(x) \right], \lambda(x) \right] + \dots \quad (4.10)$$

and chose λ via the solution of the equation

$$\frac{\partial \lambda(x)}{\partial x_\mu} + \mathcal{D}_\mu(x) = -\frac{1}{\Delta} \frac{\partial}{\partial x_\mu} \frac{\partial A_o^L(x)}{\partial x_o} \quad (4.11)$$

Then we get

$$A_\mu^C(x) = A_\mu^L(x) - \frac{1}{\Delta} \frac{\partial}{\partial x_\mu} \frac{\partial A_o^L(x)}{\partial x_o} \quad (4.12)$$

which is valid also in the classical physic, where λ commute with A_μ and $\mathcal{D}_\mu = 0$.

It is easy to check that A_μ^C (4.12) satisfy the gauge conditions (3.1), the equation of motion (3.3) and the Poisson equation (3.5). Moreover, Ψ^C after gauge transformation (4.9) satisfies the equation of motion (3.4).

Substitution A_μ^C (4.12) in (3.5) yields

$$-\Delta A_o^C = \square A_o^L = J_o(x) \quad (4.13)$$

which produces the following relationship between the nucleon form factors in the Lorentz and Coulomb gauges

$$t_N < out; \mathbf{p}'_N | A_o^L(0) | \mathbf{p}_N; in > = -(\mathbf{p}'_N - \mathbf{p}_N)^2 < out; \mathbf{p}'_N | A_o^C(0) | \mathbf{p}_N; in > \quad (4.14)$$

The relation (4.12) allows equate the leading one photon exchange potential V_{OPE} (2.7b) in the Lorentz and Coulomb gauges

$$V_{OPE}^L = V_{OPE}^C \quad (4.15a)$$

where

$$V_{OPE}^L = e \bar{u}(\mathbf{p}'_e) \gamma_\mu u(\mathbf{p}_e) < out; \mathbf{p}'_N | A_\mu^L(0) | \mathbf{p}_N; in > WY, \quad (4.14b)$$

$$V_{OPE}^C = e \bar{u}(\mathbf{p}'_e) \gamma_\mu u(\mathbf{p}_e) < out; \mathbf{p}'_N | A_\mu^C(0) | \mathbf{p}_N; in >, \quad (4.14c)$$

because

$$\begin{aligned} & \bar{u}(\mathbf{p}'_e) \gamma_\mu u(\mathbf{p}_e) < out; \mathbf{p}'_N | \left[\frac{\partial}{\partial x_\mu} \frac{\partial A_o^L(x)}{\partial x_o} \right]_{x=0} | \mathbf{p}_N; in > = \\ & -i \bar{u}(\mathbf{p}'_e) \gamma_\mu (p'_N - p_N)^\mu u(\mathbf{p}_e) < out; \mathbf{p}'_N | \left[\frac{\partial A_o^L(x)}{\partial x_o} \right]_{x=0} | \mathbf{p}_N; in > = 0 \end{aligned} \quad (4.16)$$

The canonical equal time anti commutator of the electron fields in the Lorentz gauge

$$\left\{ \Psi^{L+}(x_o, \mathbf{x}), \Psi^L(x_o, \mathbf{y}) \right\} = \delta(\mathbf{x} - \mathbf{y}) \quad (4.17a)$$

after gauge transformation (4.9) produces the following canonical anti commutators in the Coulomb gauge

$$\left\{ \Psi^{C+}(x_o, \mathbf{x}), \Psi^C(x_o, \mathbf{y}) \right\} = \delta(\mathbf{x} - \mathbf{y}) + \mathcal{C}(x, \mathbf{y}), \quad (4.17b)$$

where

$$\begin{aligned} \mathcal{C}(x, \mathbf{y}) = & \Psi^{L+}(x) \left[e^{-i\lambda(x)}, e^{i\lambda(x_o, \mathbf{y})} \right] \Psi^L(x_o, \mathbf{y}) \\ & + \left[\Psi^{L+}(x), e^{i\lambda(x_o, \mathbf{y})} \right] e^{-i\lambda(x_o, \mathbf{y})} \Psi^L(x_o, \mathbf{y}) + e^{i\lambda(x_o, \mathbf{y})} \left[\Psi^L(x_o, \mathbf{y}), e^{-i\lambda(x_o, \mathbf{y})} \right] \Psi^{L+}(x) \end{aligned} \quad (4.18)$$

The anti commutator (4.17) and the gauge transformation (4.9) generate the additional terms with \mathcal{D}_μ (4.10) and \mathcal{C}_μ (4.18) in Y_I^C (3.10b) and Y_{II}^C (3.10c) which gives also the small corrections in order to e^5 .

5. Conclusion

In this paper the general time ordered field-theoretical unitarity condition (2.5) are used for derivation of the 3D relativistic Lippmann-Schwinger type equations (2.13) for the ep scattering amplitude $\bar{u}(\mathbf{p}'_e) < out; \mathbf{p}'_N | \eta(0) | \mathbf{p}_e \mathbf{p}_N; in >$ (2.1a). In the unitarity condition (2.5) and the

corresponding equation (2.13) the nucleons are on mass shell and only one of the external electrons is off mass shell. Therefore in this approach the proton self energy diagrams, the proton vertex correction and other diagrams with the off mass shell nucleons do not appear. The leading order part of the potential of the equation (2.13) is the one photon exchange potential V_{OPE} (2.7b) in Fig. 1 which is constructed via the usual one variable electromagnetic form factors of the nucleon (2.9).

The unitarity condition (2.5) follows directly from the \mathcal{S} -matrix reduction formula for the ep scattering amplitude $\mathcal{A}_{e'N',eN}$ (2.1a). Consequently, unitarity condition (2.5) and its representation (2.13) are the necessary conditions for the ep scattering amplitude in QED . Moreover, the \mathcal{S} -matrix reduction technique provides us with the analytic relations between the $\mathcal{A}_{e'N',eN}$ (2.1a) and the corresponding 4D amplitude of the Bethe-Salpeter equation and their 3D quasipotential reductions. Therefore any results obtained in the present approach can be reproduced within the 4D formulation and vice versa.

It is shown that the leading order potential V_{OPE} (2.7b) together with the additional terms (3.10c) are generated by the canonical equal time anti commutators (2.7a). These anti commutators are not invariant under the gauge transformations of the commuting fields. Nevertheless it is demonstrated that V_{OPE} is the same in the Coulomb and Lorentz gauge. The additional terms (3.10c) and (3.12a)-(3.12d) are produced by the electrostatic (Coulomb) interactions and they give small corrections in order of $\sim e^5$. The Coulomb gauge is more convenient for calculation of these corrections.

The unitarity condition (2.5) contains only the hadron and lepton degrees of freedom due to the equal time commutators of the electron fields (2.7a) and the completeness condition of the asymptotic particle states (2.4). Therefore the quark-gluon degrees of freedom can contribute in (2.5) and (2.13) only through the input electromagnetic nucleon vertex $\langle out; \mathbf{p}' | J_\mu(0) | \mathbf{p}; in'' \rangle$ (2.9).

The alternative unitarity condition can be obtained using the \mathcal{S} -matrix reduction formulas for the on mass shell electrons and off mass shell nucleons. The composed nucleon field can be constructed within the Haag-Nisijima-Zimmermann approach [26, 27, 28, 25]. In this case the equal time commutators for the composed nucleon fields [15, 16, 19] as the quark bound state operators contains all contributions from the quark-gluon degrees of freedom and the corresponding Lippmann-Schwinger type equation generates the electromagnetic corrections of this vertex.

Appendix A. Linearization of the generalized unitarity condition (2.6).

Separation of the connected and disconnected parts of the amplitudes in two last terms of (2.5) yields

$$W_{e'N',eN}(n) = (2\pi)^3 \sum_n \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}_e}(0) | n; in \rangle_C \frac{\delta(\mathbf{P}_n - \mathbf{p}_e - \mathbf{p}_N)}{P_o - P_n^o + i\epsilon} \langle in; n | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle_C \quad (A.1a)$$

$$-(2\pi)^3 \sum_{m=\gamma N, \gamma\gamma N, \dots} \langle 0 | \eta_{\mathbf{p}'_e}(0) | m; in \rangle \frac{\delta^{(3)}(\mathbf{p}_e + \mathbf{p}_2 - \mathbf{P}_m - \mathbf{p}'_2)}{P_o - P_m^o - E_{\mathbf{p}'_2} + i\epsilon} \langle in; \mathbf{p}'_2, m | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_2; in \rangle_c \quad (A.1b)$$

$$-(2\pi)^3 \sum_{m=\gamma N, \gamma\gamma N, \dots} \langle in; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_2, m; in \rangle_c \frac{\delta^{(3)}(\mathbf{p}_e - \mathbf{P}_m)}{E_{\mathbf{p}_e} - P_m^o} \langle in; m | \bar{\eta}_{\mathbf{p}_e}(0) | 0 \rangle \quad (A.1c)$$

$$-(2\pi)^3 \sum_{\bar{n}=e'\bar{N}, \gamma e'\bar{N}, \dots} \langle 0 | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_2 \bar{n}; in \rangle \frac{\delta^{(3)}(\mathbf{p}_e - \mathbf{p}'_2 - \mathbf{P}_{\bar{n}})}{E_{\mathbf{p}_e} - E_{\mathbf{p}'_2} - P_{\bar{n}}^o} \langle in; \mathbf{p}'_N \bar{n} | \bar{\eta}_{\mathbf{p}_e}(0) | 0 \rangle \quad (A.1d)$$

$$-(2\pi)^3 \sum_{v=\bar{e}N, \gamma\bar{e}N, \dots} \langle out; \mathbf{p}'_N | \bar{\eta}_{\mathbf{p}_e}(0) | v; in \rangle_C \frac{\delta(\mathbf{P}_v + \mathbf{p}_e - \mathbf{p}'_N)}{-E_{\mathbf{p}'_N} + E_{\mathbf{p}_e} + P_v^o} \langle in; v | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_N; in \rangle_C, \quad (A.1e)$$

$$+(2\pi)^3 \sum_{\bar{m}=\bar{e}, \gamma\bar{e}, \dots} \langle 0 | \bar{\eta}_{\mathbf{p}_e}(0) | \bar{m}; in \rangle \frac{\delta(\mathbf{P}_{\bar{m}} + \mathbf{p}_e)}{E_{\mathbf{p}_e} + P_{\bar{m}}^o} \langle in; \mathbf{p}'_N \bar{m} | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_N; in \rangle_C, \quad (A.1f)$$

$$+(2\pi)^3 \sum_{\bar{m}=\bar{e}, \gamma\bar{e}, \dots} \langle out; \mathbf{p}'_N | \bar{\eta}_{\mathbf{p}_e}(0) | \bar{m} \mathbf{p}_N; in \rangle_C \frac{\delta(\mathbf{P}_{\bar{m}} + \mathbf{p}_e + \mathbf{p}_N - \mathbf{p}'_N)}{-E_{\mathbf{p}'_N} + E_{\mathbf{p}_e} + E_{\mathbf{p}_N} + P_{\bar{m}}^o} \langle in; \bar{m} | \eta_{\mathbf{p}'_e}(0) | 0 \rangle, \quad (A.1g)$$

$$+(2\pi)^3 \sum_{\bar{n}=\bar{e}N, \gamma\bar{e}N, \dots} \langle 0 | \bar{\eta}_{\mathbf{p}_e}(0) | \bar{n} \mathbf{p}_N; in \rangle \frac{\delta(\mathbf{P}_{\bar{n}} + \mathbf{p}_e + \mathbf{p}_N)}{E_{\mathbf{p}_e} + E_{\mathbf{p}_N} + P_{\bar{n}}^o} \langle in; \bar{n} \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | 0 \rangle, \quad (A.1h)$$

where n denotes the complete set of the s -channel intermediate states $n = H, eN, \gamma eN, \gamma\gamma eN, \gamma H, \dots$. The index C indicates the connected part of the related amplitude and the factor \pm of the expressions (A.1b)-(A.1h) appears after transposition of the fermion fields of the electron and nucleons.

$\mathcal{W}_{e'N',eN}(n)$ without s -channel H and eN exchange terms forms the inhomogeneous part $\mathcal{W}_{e'N',eN}$ of the relation (2.10)

$$\mathcal{W}_{e'N',eN}(n = \gamma eN, \gamma \gamma eN, \gamma H \dots) = \mathcal{W}_{e'N',eN} \quad (\text{A.2})$$

$\mathcal{W}_{e'N',eN}$ with the minimal number of the intermediate states are depicted in Fig. 1A - Fig. 1H correspondingly. It is easy to see, that The last four terms in Fig. 1E-Fig. 1H are obtained from the terms in Fig. 1A-Fig. 1D by crossing of the electrons.

After simple algebra for the propagators in (A.1a)-(A.1h) we get

$$\mathcal{W}_{e'N',eN}(\mathbf{p}', \mathbf{p}) = \mathbf{A}_{e'N',eN}(\mathbf{p}', \mathbf{p}) + P'_o \mathbf{B}_{e'N',eN}(\mathbf{p}', \mathbf{p}), \quad (\text{A.3})$$

where $\mathbf{A}_{e'N',eN}$ and $\mathbf{B}_{e'N',eN}$ are the following Hermitian matrices

$$\begin{aligned} \mathbf{A}_{e'N',eN} &= (2\pi)^3 \sum_{n=\gamma eN, \gamma \gamma eN, \gamma H \dots} \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | n; in \rangle_C \frac{(-P'_o) \delta(\mathbf{P}_n - \mathbf{p}_e - \mathbf{p}_N)}{P'_o - P'_n + i\epsilon} \frac{\langle in; n | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle_C}{P'_o - P'_n - i\epsilon} \\ &- (2\pi)^3 \sum_{m=\gamma N, \gamma \gamma N, \dots} \langle 0 | \eta_{\mathbf{p}'_e}(0) | m; in \rangle \frac{(-P'_o - E_{\mathbf{p}'_2}) \delta^{(3)}(\mathbf{p}_e + \mathbf{p}_2 - \mathbf{P}_m - \mathbf{p}'_2)}{P'_o - P'_m - E_{\mathbf{p}'_2} + i\epsilon} \frac{\langle in; \mathbf{p}'_2, m | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_2; in \rangle_c}{P'_o - P'_m - E_{\mathbf{p}'_2}} \\ &- (2\pi)^3 \sum_{m=\gamma N, \gamma \gamma N, \dots} \langle in; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_2, m; in \rangle_c \frac{(-P'_o - E_{\mathbf{p}_2}) \delta^{(3)}(\mathbf{p}_e - \mathbf{P}_m)}{E_{\mathbf{p}_e} - P'_m} \frac{\langle in; m | \bar{\eta}_{\mathbf{p}_e}(0) | 0 \rangle}{P'_o - P'_m - E_{\mathbf{p}_2} - i\epsilon} \\ &= (2\pi)^3 \sum_{\bar{n}=e''N, \gamma e''N \dots} \langle 0 | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_2 \bar{n}; in \rangle \frac{(-E_{\mathbf{p}_N} - E_{\mathbf{p}'_2} - P'_o) \delta^{(3)}(\mathbf{p}_e - \mathbf{p}'_2 - \mathbf{P}_{\bar{n}})}{E_{\mathbf{p}_e} - E_{\mathbf{p}'_2} - P'_o} \frac{\langle in; \mathbf{p}'_N \bar{n} | \bar{\eta}_{\mathbf{p}_e}(0) | 0 \rangle}{P'_o - E_{\mathbf{p}_N} - E_{\mathbf{p}'_2} - P'_o} \\ &\quad + \text{electron crossing terms} \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \mathbf{B}_{e'N',eN} &= (2\pi)^3 \sum_{n=\gamma eN, \gamma \gamma eN, \gamma H \dots} \frac{\langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | n; in \rangle_C}{P'_o - P'_n - i\epsilon} \delta(\mathbf{P}_n - \mathbf{p}_e - \mathbf{p}_N) \frac{\langle in; n | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle_C}{P'_o - P'_n + i\epsilon} \\ &- (2\pi)^3 \sum_{m=\gamma N, \gamma \gamma N, \dots} \frac{\langle 0 | \eta_{\mathbf{p}'_e}(0) | m; in \rangle}{P'_o - P'_m - E_{\mathbf{p}'_2}} \delta^{(3)}(\mathbf{p}_e + \mathbf{p}_2 - \mathbf{P}_m - \mathbf{p}'_2) \frac{\langle in; \mathbf{p}'_2, m | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_2; in \rangle_c}{P'_o - P'_m - E_{\mathbf{p}'_2} + i\epsilon} \\ &- (2\pi)^3 \sum_{m=\gamma N, \gamma \gamma N, \dots} \frac{\langle in; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_2, m; in \rangle_c}{P'_o - P'_m - E_{\mathbf{p}_2} - i\epsilon} \delta^{(3)}(\mathbf{p}_e - \mathbf{P}_m) \frac{\langle in; m | \bar{\eta}_{\mathbf{p}_e}(0) | 0 \rangle}{E_{\mathbf{p}_e} - P'_m} \end{aligned}$$

$$\begin{aligned}
&= (2\pi)^3 \sum_{\bar{n}=e''\bar{N}, \gamma e''\bar{N} \dots} \frac{\langle 0 | \eta_{\mathbf{p}'_e}(0) | \mathbf{p}_2 \bar{n}; in \rangle}{E_{\mathbf{p}'_e} - E_{\mathbf{p}_N} - P_{\bar{n}}^0} \delta^{(3)}(\mathbf{p}_e - \mathbf{p}'_2 - \mathbf{p}_{\bar{n}}) \frac{\langle in; \mathbf{p}'_N \bar{n} | \bar{\eta}_{\mathbf{p}_e}(0) | 0 \rangle}{E_{\mathbf{p}_e} - E_{\mathbf{p}'_2} - P_{\bar{n}}^0} \\
&\quad + \text{electron crossing 4 terms}
\end{aligned} \tag{A.5}$$

The Hermitian matrices $\mathbf{A}_{e'N',eN}$ (A.4) and $\mathbf{B}_{e'N',eN}$ (A.5) together with the equal-time commutator $Y_{e'N',eN}$ (2.7a) form the linear energy depending potential $\mathcal{U}(\mathbf{p}', \mathbf{q}, E)$ of the relativistic Lippmann-Schwinger equation (2.13)

$$\mathcal{U}(\mathbf{p}', \mathbf{q}, E) \equiv \mathcal{U}(E) = Y_{e'N',eN} + \mathbf{A}_{e'N',eN} + E \mathbf{B}_{e'N',eN} \tag{A.6}$$

It must be noted that one can determinate $\mathbf{B}_{e'N',eN}$ (A.5) through the s channel terms with the H and eN intermediate states

$$\begin{aligned}
&\delta^{(3)}(\mathbf{P}' - \mathbf{P}) \left[(2\pi)^3 \sum_{n=H; e''N''} \langle out; \mathbf{p}'_N | \eta_{\mathbf{p}'_e}(0) | n; in \rangle_C \frac{\delta(\mathbf{P}_n - \mathbf{p}_e - \mathbf{p}_N)}{P_o - P_n^0 + i\epsilon} \langle in; n | \bar{\eta}_{\mathbf{p}_e}(0) | \mathbf{p}_N; in \rangle_C \right. \\
&\quad \left. + \mathbf{B}_{e'N',eN} \right] = \langle in; \mathbf{p}'_N | \{ b_{\mathbf{p}_e}(0), b_{\mathbf{p}_e}^+(0) \} | \mathbf{p}_N; in \rangle = \langle in; \mathbf{p}'_e \mathbf{p}'_N | \mathbf{p}_e \mathbf{p}_N; in \rangle
\end{aligned} \tag{A.7}$$

Therefore, the operator $(1 - B)$ corresponds to the contributions of the intermediate s channel eN and H states in the commutation relation for the Heisenberg field operators.

Using the same technique as for the other binary reactions in [13, 14, 15, 16, 17]. one can derive (2.5) from the Lippmann-Schwinger-type equation (2.13) and vice verse

References

- [1] J. Arrington, P. G. Blunden and W. Melnitchouk, Prog. Part. Nucl. Phys. **66** (2011) 782.
- [2] N. Kivel and M. Vanderhaeghen, JHEP, **04** (2013) 029.
- [3] C. E. Perdrisat, V. Punjabi and Vanderhaeghen, Prog. Part. Nucl. Phys. 59 (2007) 694.
- [4] J. Arrington, W. Melnitchouk and J.A. Tjon. Phys. Rev. **C76** (2007) 035205.
- [5] C. Itzykson and C. Zuber. Quantum Field Theory. (New York, McGraw-Hill) 1980.
- [6] S. Weinberg. The Quantum Theory of Fields. (Cambridge, Univesity Press), vol.II 2001.
- [7] F.J.Dyson, Phys. Rev **85** (1952) 631.
- [8] I.M.Suslov, JETP **100** (2005) 1188.

- [9] M. Eades et al; Phys. Rep. **342** (2001) 66.
- [10] S. G. Karschenboim; Phys. Rep. **422** (2005) 1.
- [11] G. T. Bodwin, D. R. Yennie and M. A. Gregorio; Rev. Mod. Phys. **57** (1985) 723.
- [12] J. Eichler, Phys. Rep. **193**(1990) 165.
- [13] A. I. Machavariani and A.G. Rusetsky, Nucl. Phys. **A515**(1990) 671.
- [14] A. I. Machavariani and A.G.Chelidze, J. Phys. **G9**(1993) 128.
- [15] A. I. Machavariani, Phys. Part. Nucl. (Sov. J. Part. Nucl.) **24(3)** (1993) 317.
- [16] A. I. Machavariani, A. Buchmann and Amand Faessler, Nucl. Phys. **A646** (1999) 231.
- [17] A. I. Machavariani and Amand Faessler, Ann. of Phys. **409** (2004) 149.
- [18] A. I. Machavariani and Amand Faessler, Phys. Review **C72** (2005) 024002.
- [19] A. I. Machavariani and Amand Faessler, Jour. Phys. G. **37** (2010) 75004; *ibid.* **38** (2011) 035002.
- [20] J. D. Bjorken and S.D.Drell, Relativistic Quantum Fields. (New York, McGraw-Hill) 1965.
- [21] S. Weinberg. The Quantum Theory of Fields. (Cambridge, Univesity Press), vol.I 2000.
- [22] V. De Alfaro, S. Fubini, G. Furlan and C. Rosseti, Currents in Hadron Physics (North-Holland, Amsterdam) 1973.
- [23] G. F. Chew and F. Low, Phys. Rev. **101** (1956) 1579.
- [24] M. K. Banerjee and J. B. Cammarata, Phys. Rev. **C17** (1978) 1125.
- [25] K. Huang and H. A. Weldon, Phys. Rev. **D11** (1975) 257.
- [26] R. Haag, Phys. Rev. **112** (1958) 669.
- [27] K. Nishijima, Phys. Rev. **111** (1958) 995.
- [28] W. Zimmermann, Nuovo Cim. **10** (1958) 598.